647510 **MSC2MatC204x** **Seat No :________**

M.Sc. (Maths) Semester - 2 (*CBCS*) Examination
August/September -2020 [NEW COURSE]
METHODS IN PARTIAL DIFFERENTIAL EQUATIONS – 2 (CORE)

Time: 2:00 Hours **Marks: 56**

Instructions:

1. Figure to the right indicate marks.
2. There are five questions in the question paper.
3. Answer any four of the following questions.

Q-1) Answer any seven: [7 x 2=14]

1. Find the complete integral of $z = p^3 + q^3$.
2. Find the direction cosines of normal to the surface $2x + 3y + 5z = 7$ at the point (1,1,0).
3. Find the complementary function of $\left(D^2 + 3DD' - 4D'^2\right)z = 0$.
4. Eliminate the arbitrary constants a and b from $z = (ax + y)^2 + b$.
5. Write down Jacobi's auxiliary equation.
6. Check $(6x + yz)dx + (xy + 2z)dz + (xz - 2y)dy = 0$ is integrable or not?
7. Solve $F\left(x + y, x - \sqrt{z}\right) = 0$, where F is an arbitrary function.
8. Define i) Pffafian differential form and ii) Tangent plane
9. Find the envelope of the given surface $(x - a)^2 + (y - b)^2 = (z)^2 + 1$.
10. Classify the partial differential equation $4y^2 \frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0$.

Q-2) Answer any two: [2 x 7=14]

a) Solve $(p^2 + q^2)y = qz$ using charpits method.

b) Find the orthogonal trajectories on the conicoid $(x + y)z = 1$ of the conics in which it is cut by the system of planes $x - y + 2 = d$ where d is the parameter.

c) Classify the equation and convert it into canonical form $9r - 6s + t = y$.

Q-3) All are compulsory: [2 x 7=14]

a) Find the primitive solution of $yzdx + (x^2y - zx)dy + (x^2z - yx)dz = 0$ if possible.

b) Find the equation of the integral surface of the differential equation $2y(z - 3)p + (2x - z)q = y(2x - 3)$ which passes through $z = 0$ and$x^2 + y^2 = 2x$.

OR

Q-3) All are Compulsory: [2 x 7=14]

a) Find the general solution of$\cos(x + y)\, p + \sin(x + y)\, q = z$.

b) Find the general integral of the partial differential equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$ which passes through the line $x = 1$ and $y = 0$.

Q-4) Answer any two: [2 x 7=14]

a) Using Natani's method $z(y^2 + z)dx + z(x^2 + z)dy - xy(x + y)dz = 0$.

b) If $X = X(x, y, z)$ and $\mu = \mu(x, y, z)$ be two functions with $\mu \neq 0$ then,
$$X \cdot curl(X) = 0 \text{ if and only if } \mu X \cdot curl(\mu X) = 0$$

c) Solve the equation $zpq = p + q$.

Q-5) Answer any two: [2 x 7=14]

a) Solve $x^2D^2 - y^2D'^2 + xD - yD' = x - y$.

b) Prove that $F\left(D, D'\right)[e^{ax+by} \cdot h(x, y)] = e^{ax+by} F\left(D + a, D' + b\right)[h(x, y)]$.

c) Find particular integral of $\left(D - D'\right)\left(2D^2 - 7D' + 3\right)z = \sin\,(3y - x)$.

d) If $(\beta D' + \gamma)^2$ with $\beta \neq 0$ is a factor of $F(D, D')$, then a solution of the equation $F(D, D')$ is $z = e^{\frac{-\gamma}{\beta}y}\left(\emptyset_1(\beta x) + y\emptyset_2(\beta x)\right)$; where $\emptyset_i = \emptyset_i(\varepsilon)$ is an arbitrary function of a single variable $(i = 1,2)$.

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